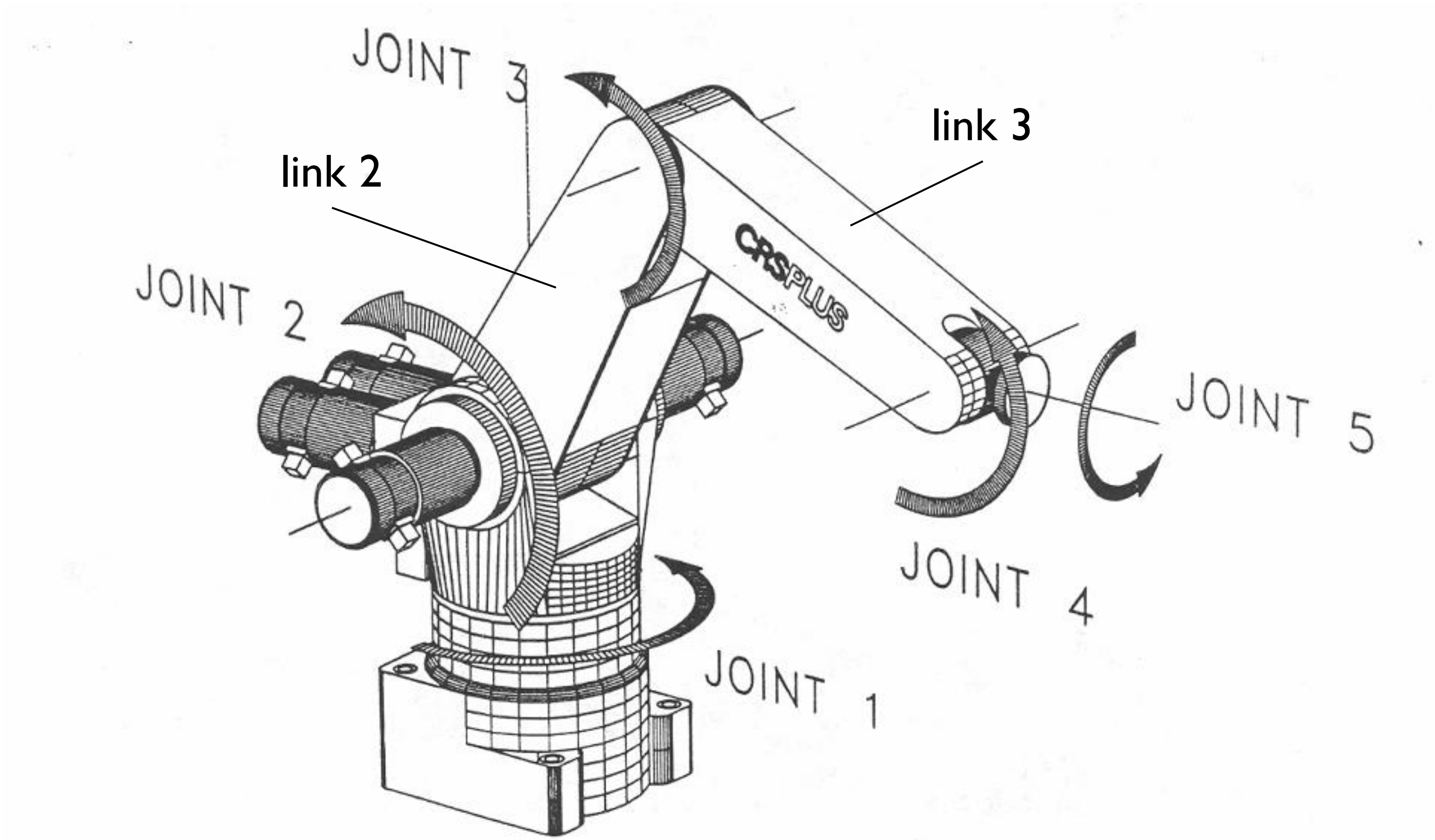
Day 02

Introduction to manipulator kinematics

Robotic Manipulators

- a robotic manipulator is a kinematic chain
  - i.e. an assembly of pairs of rigid bodies that can move respect to one another via a mechanical constraint
- the rigid bodies are called *links*
- the mechanical constraints are called *joints*

A150 Robotic Arm



Joints

- most manipulator joints are one of two types

1. revolute (or rotary)
    - like a hinge
    - allows relative rotation about a fixed axis between two links
        - axis of rotation is the z axis by convention
2. prismatic (or linear)
    - like a piston
    - allows relative translation along a fixed axis between two links
        - axis of translation is the z axis by convention

- our convention: joint i connects link $i - 1$ to link i
    - when joint i is actuated, link i moves

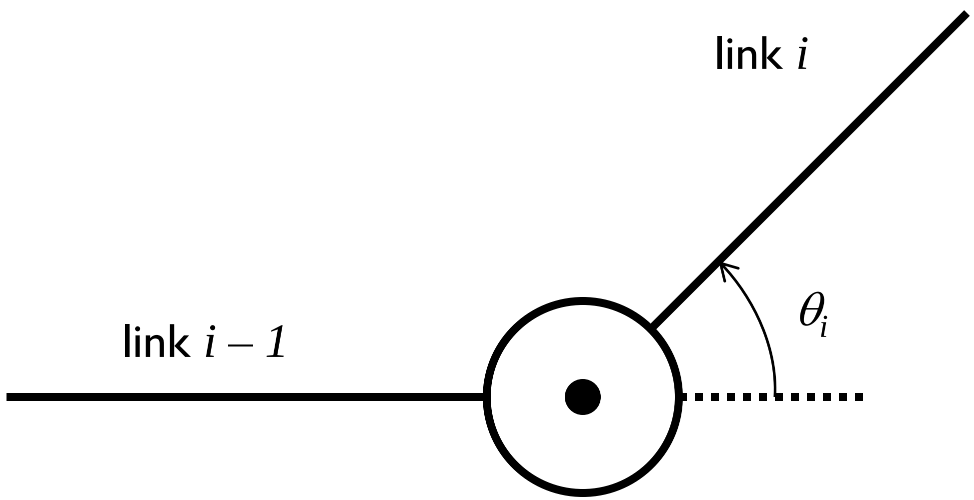
Joint Variables

- revolute and prismatic joints are one degree of freedom (DOF) joints; thus, they can be described using a single numeric value called a joint variable
- q_i : joint variable for joint i

1. revolute
   - $q_i = \theta_i$: angle of rotation of link i relative to link $i-1$
2. prismatic
   - $q_i = d_i$: displacement of link i relative to link $i-1$

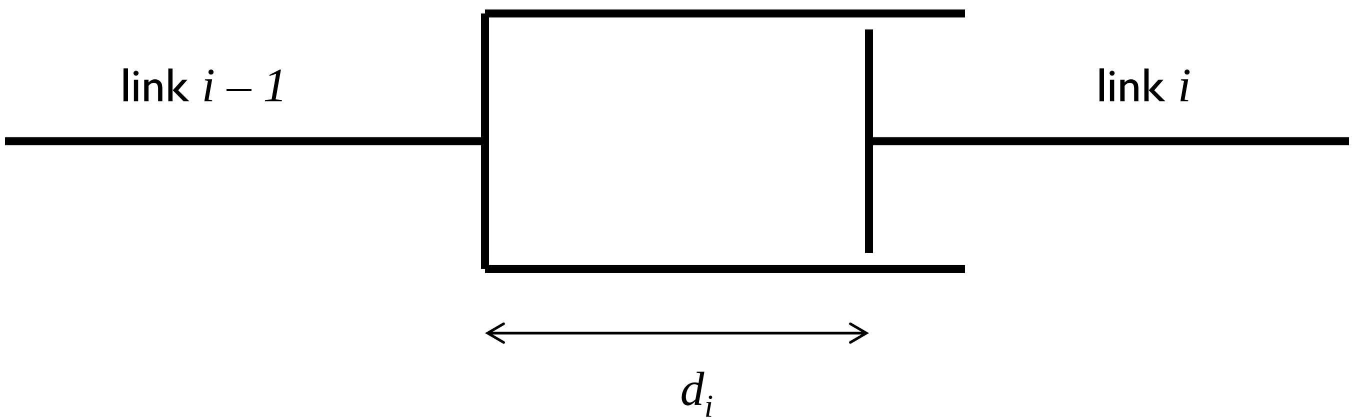
Revolute Joint Variable

- revolute
  - $q_i = \theta_i$: angle of rotation of link i relative to link $i-1$



Prismatic Joint Variable

- prismatic
  - $q_i = d_i$: displacement of link i relative to link $i-1$

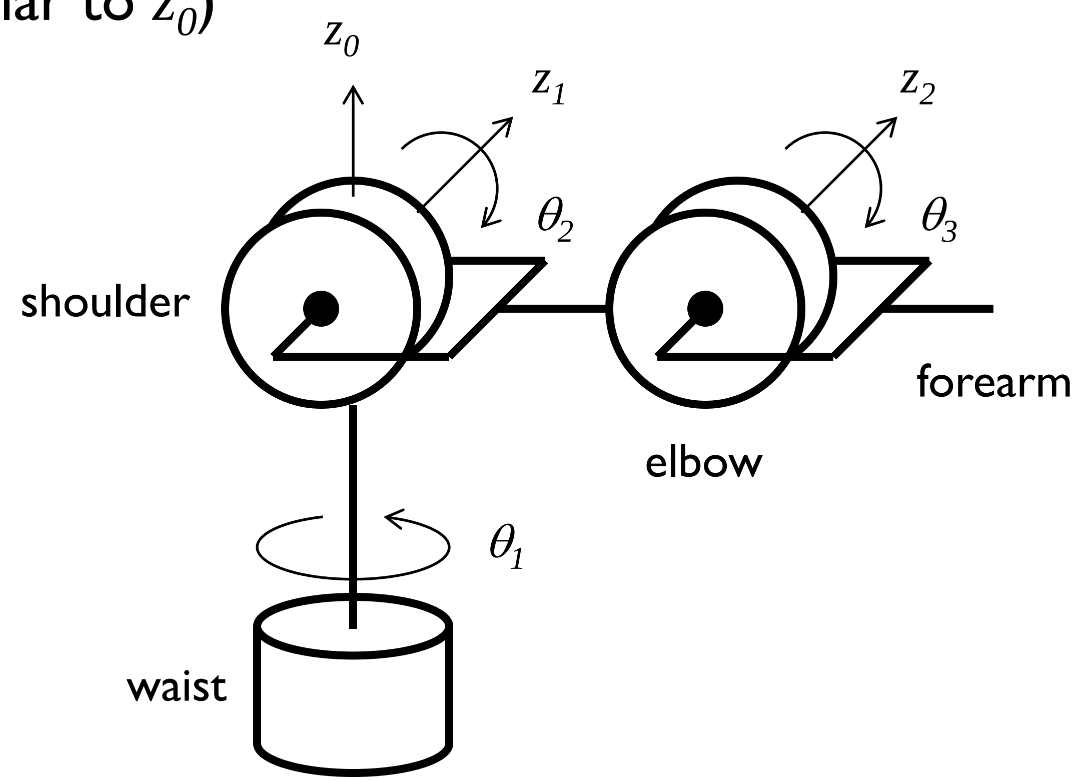


Common Manipulator Arrangments

- most industrial manipulators have six or fewer joints
  - the first three joints are the arm
  - the remaining joints are the wrist
- it is common to describe such manipulators using the joints of the arm
  - R: revolute joint
  - P: prismatic joint

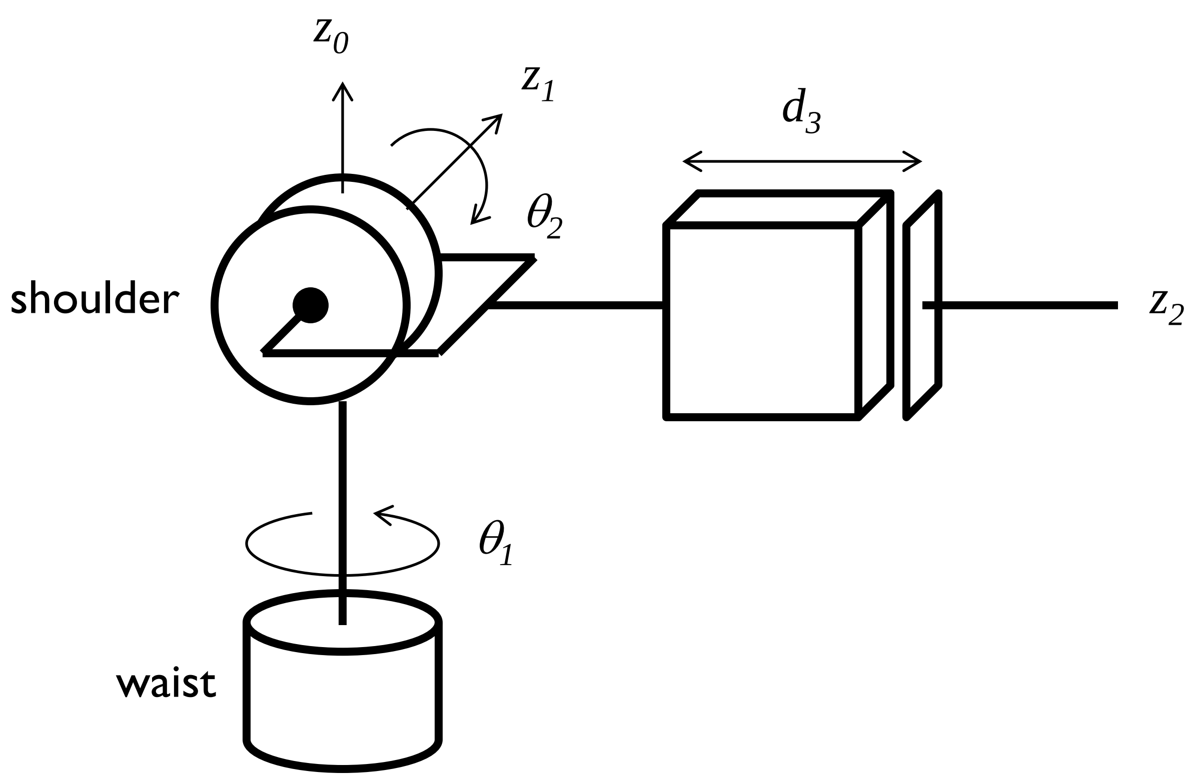
Articulated Manipulator

- RRR (first three joints are all revolute)
- joint axes
  - z_0 : waist
  - z_1 : shoulder (perpendicular to z_0)
  - z_2 : elbow (parallel to z_1)



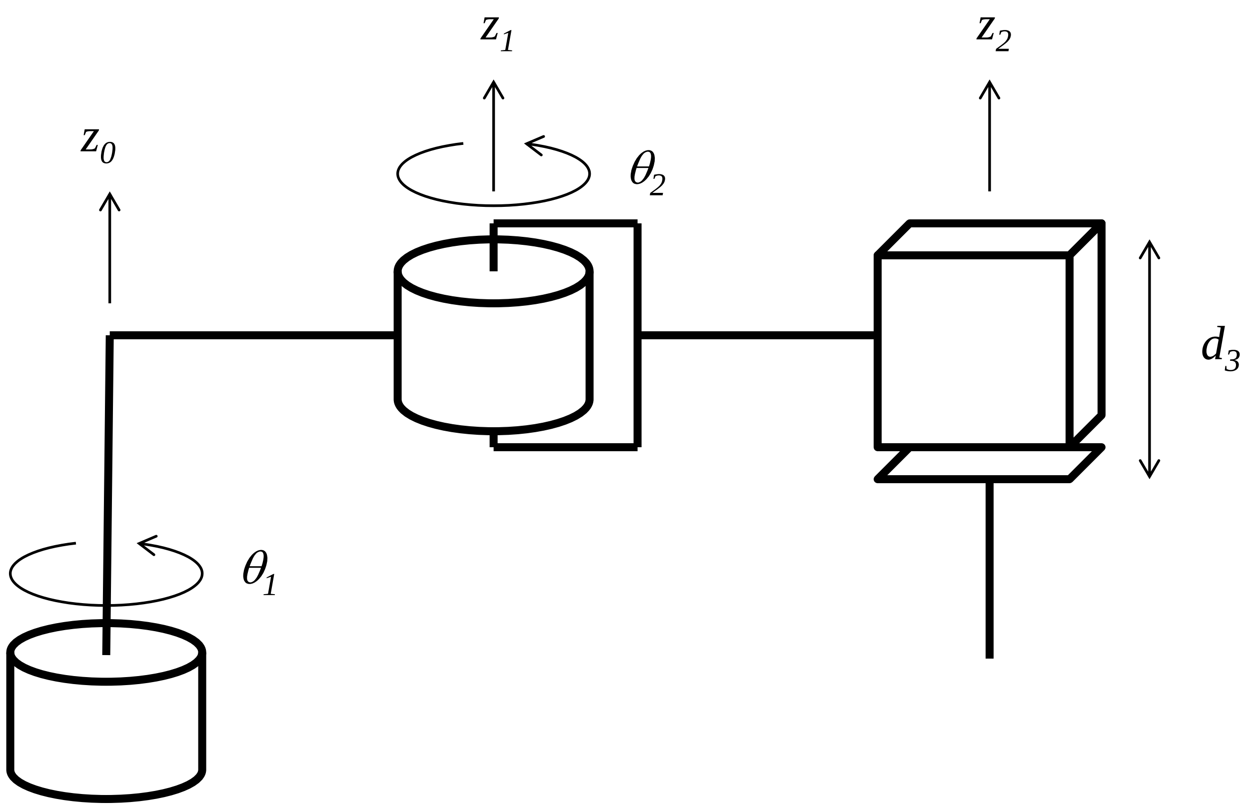
Spherical Manipulator

- RRP
- Stanford arm
  - http://infolab.stanford.edu/pub/voy/museum/pictures/display/robots/IMG_2404ArmFrontPeekingOut.JPG



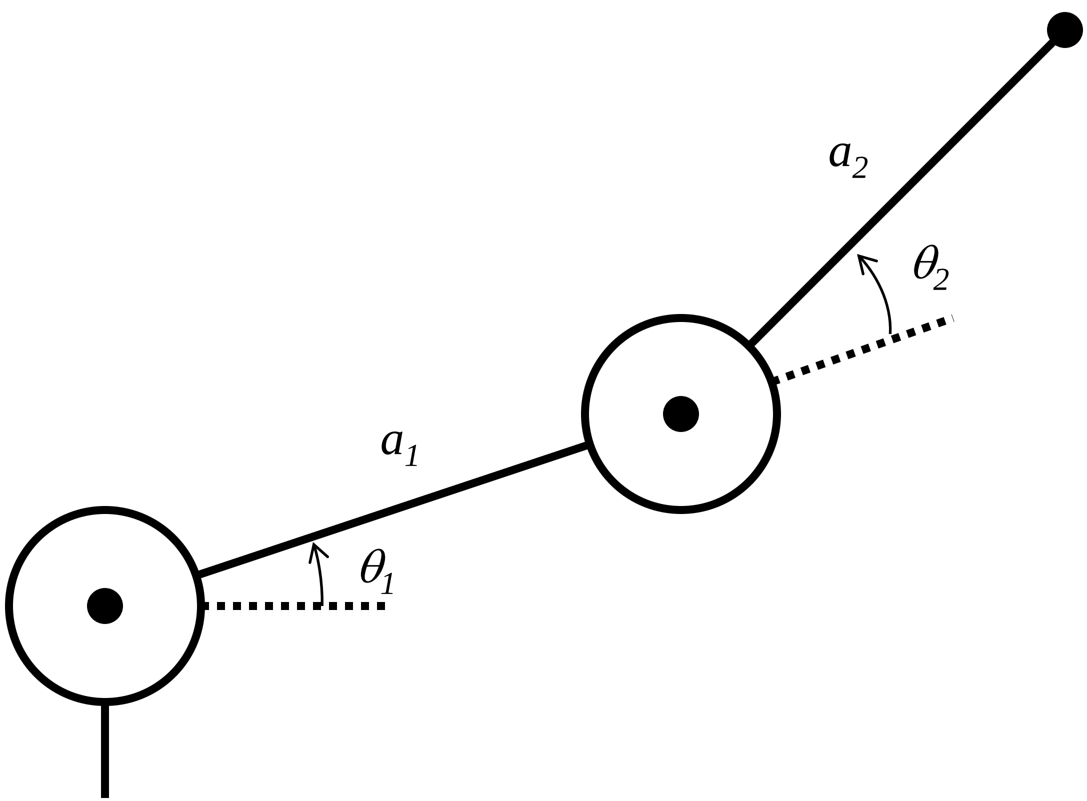
SCARA Manipulator

- RRP
- Selective Compliant Articulated Robot for Assembly
  - http://www.robots.epson.com/products/g-series.htm



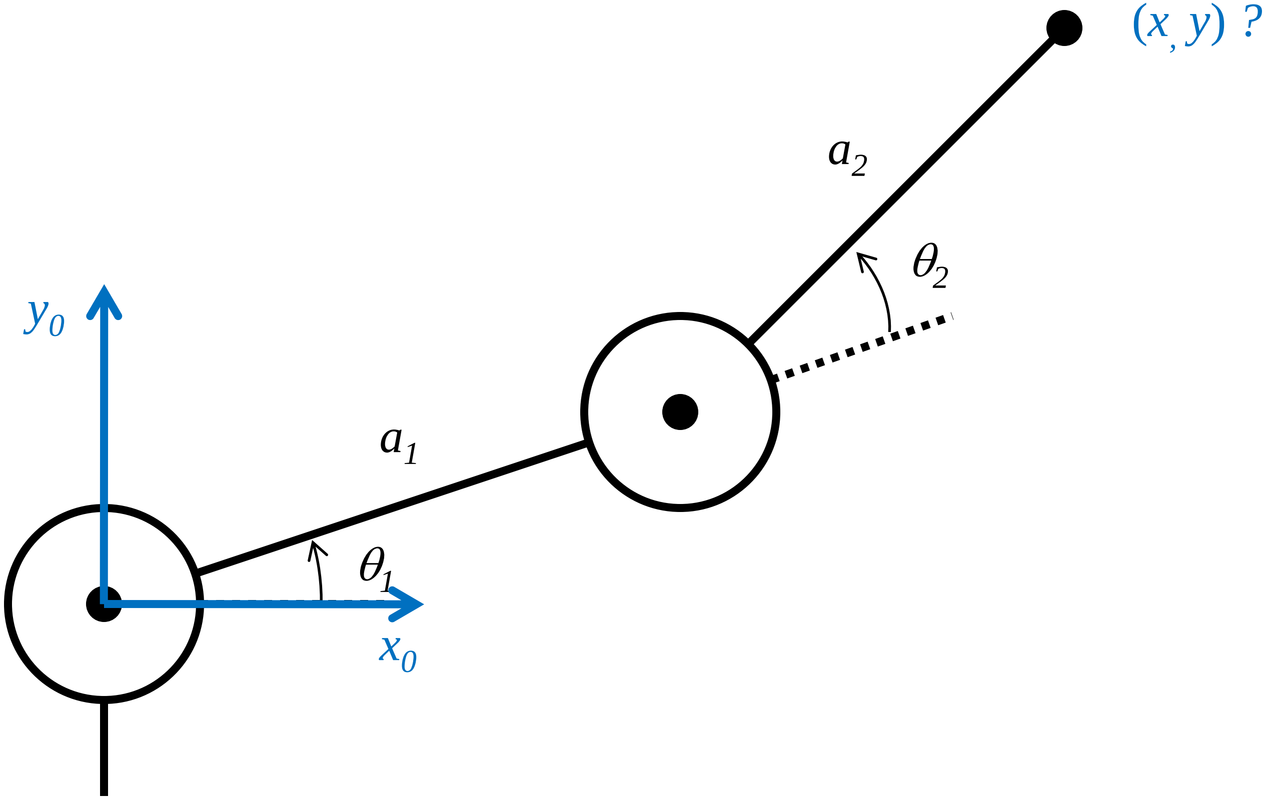
Forward Kinematics

- given the joint variables and dimensions of the links what is the position and orientation of the end effector?



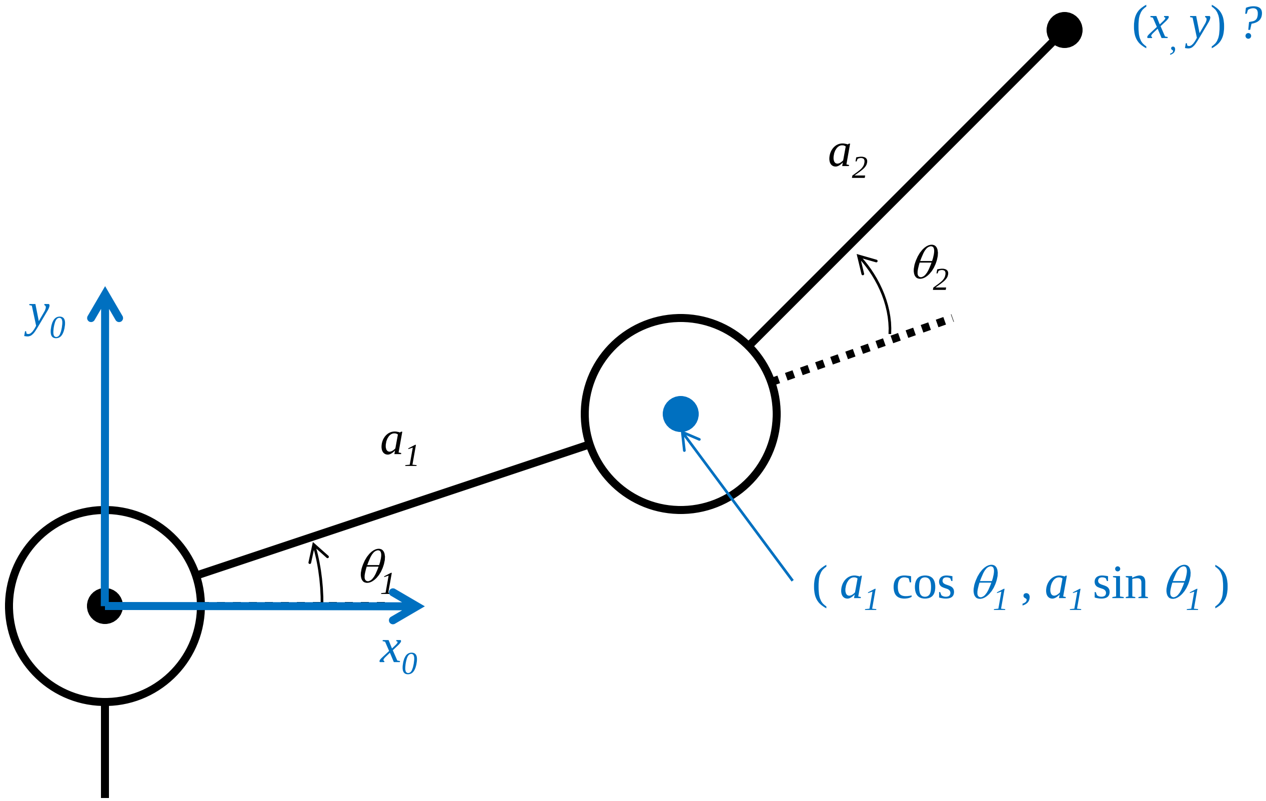
Forward Kinematics

- choose the base coordinate frame of the robot
  - we want (x, y) to be expressed in this frame



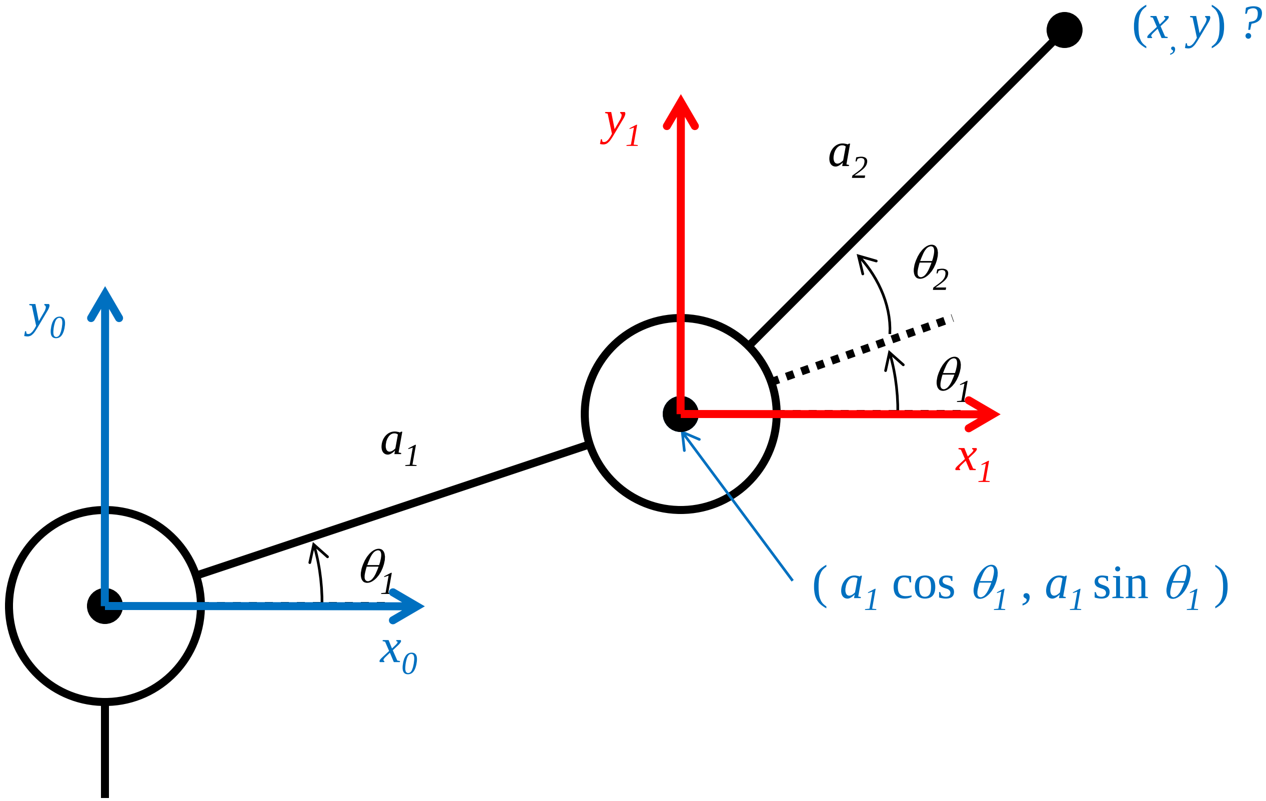
Forward Kinematics

- notice that link *1* moves in a circle centered on the base frame origin



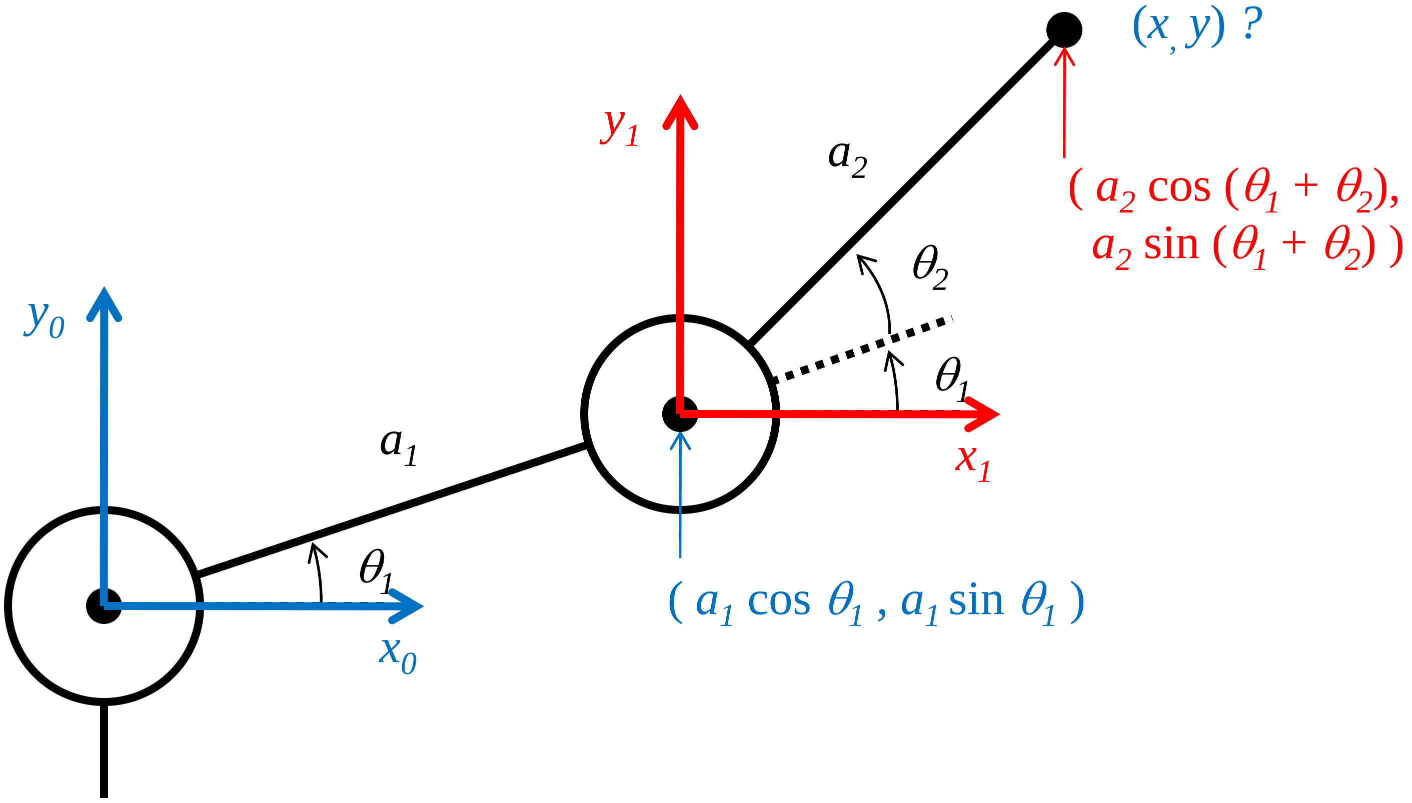
Forward Kinematics

- choose a coordinate frame with origin located on joint *2* with the same orientation as the base frame



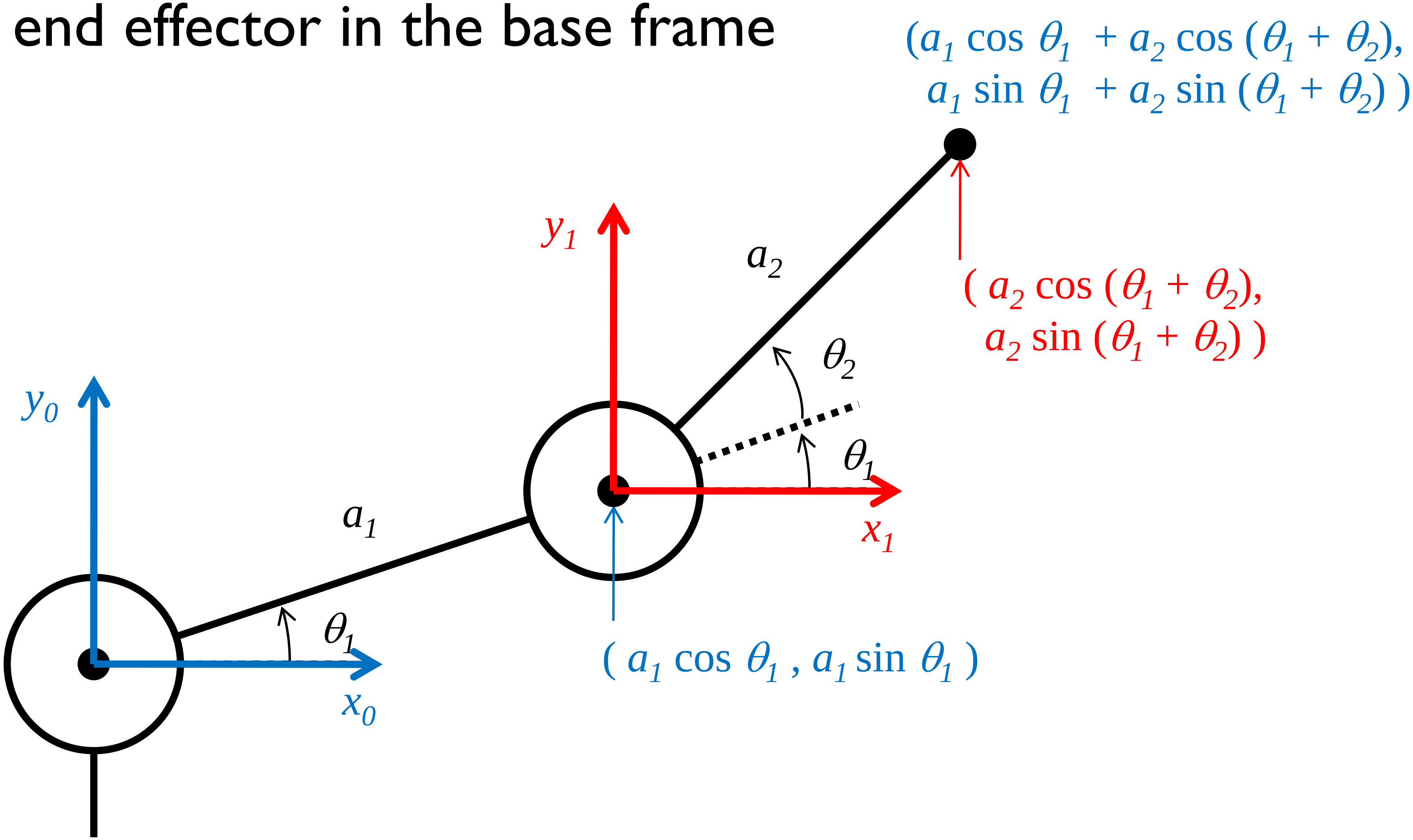
Forward Kinematics

- notice that link *2* moves in a circle centered on frame *1*



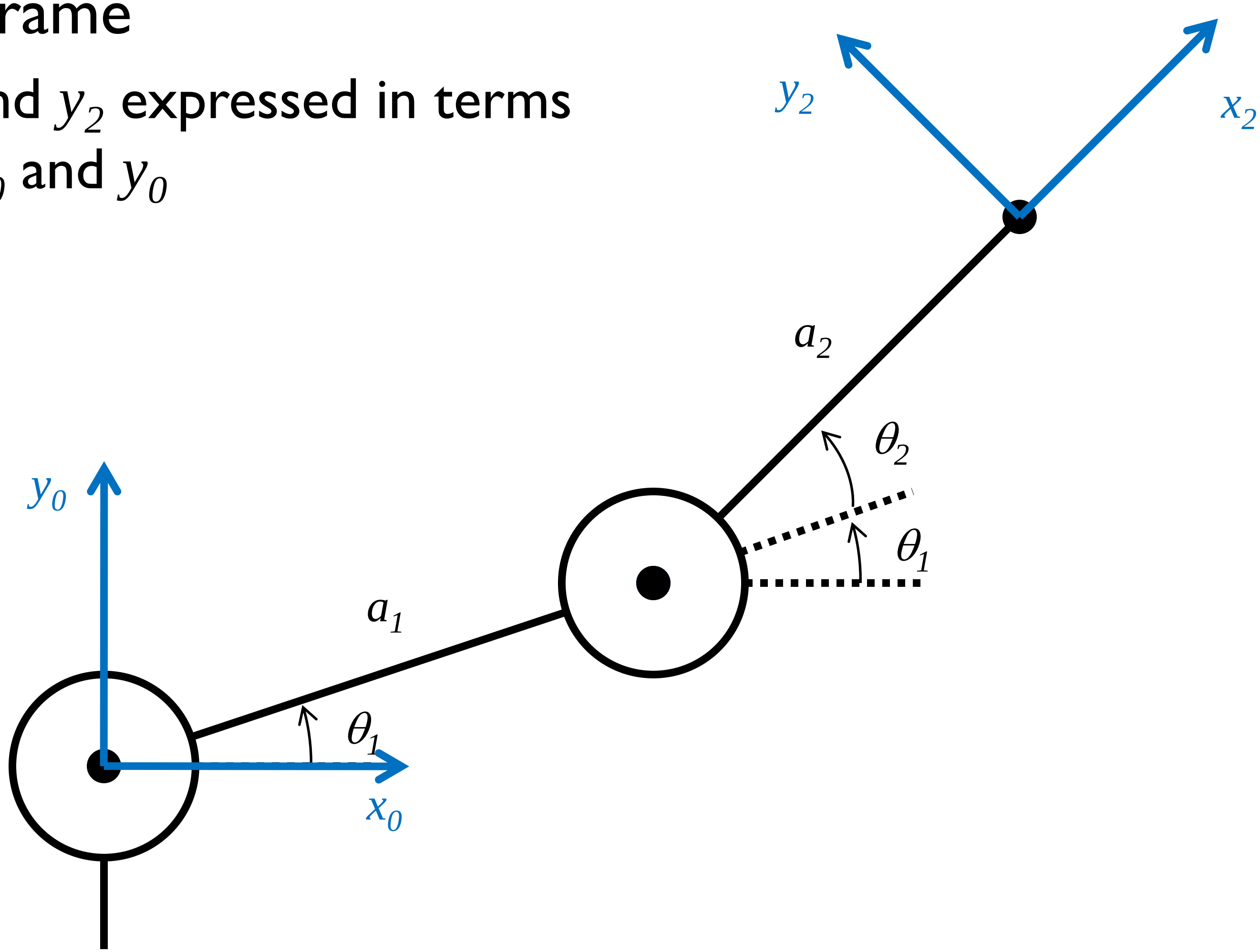
Forward Kinematics

- because the base frame and frame *1* have the same orientation, we can sum the coordinates to find the position of the end effector in the base frame



Forward Kinematics

- we also want the orientation of frame 2 with respect to the base frame
  - x_2 and y_2 expressed in terms of x_0 and y_0

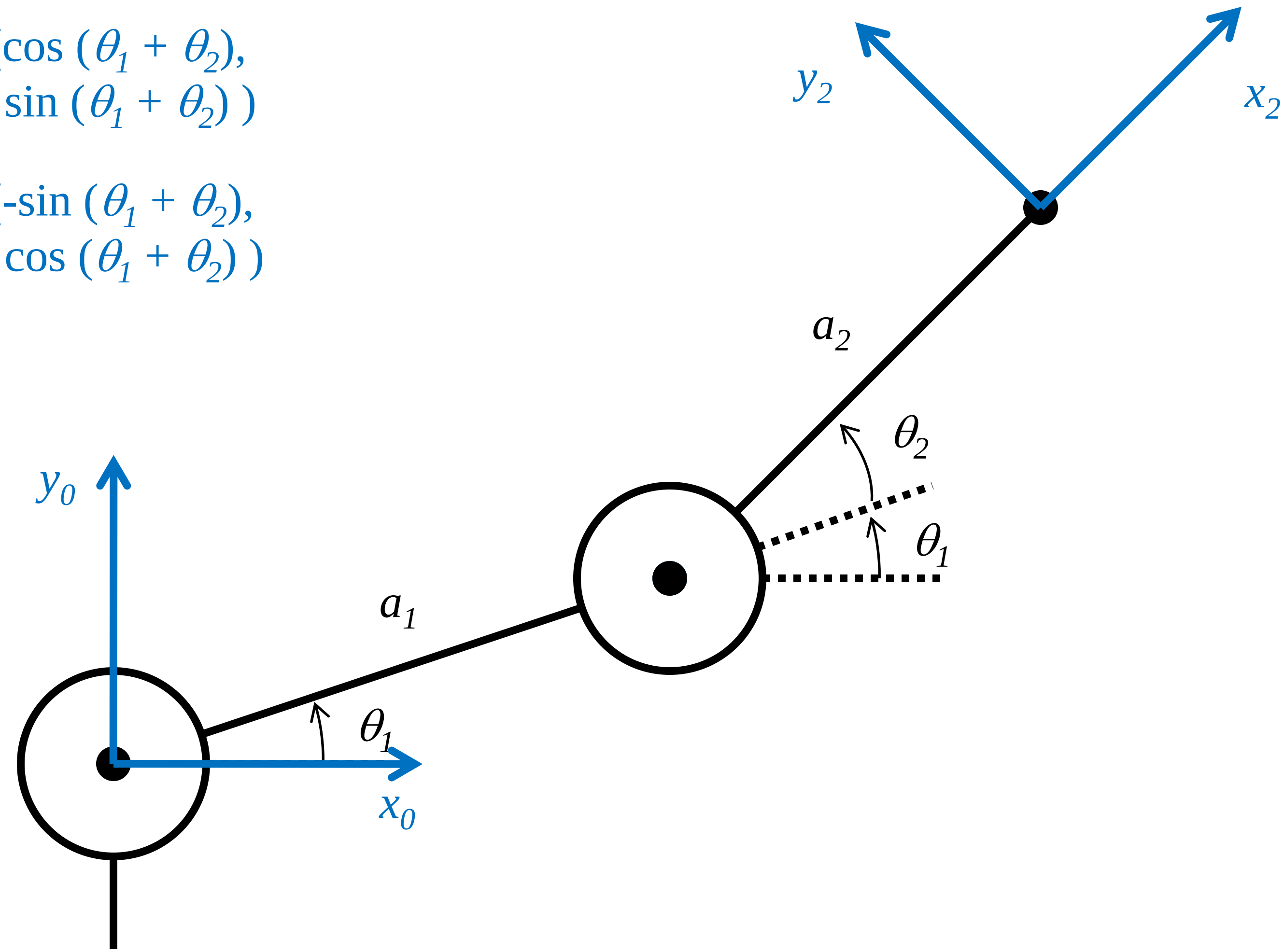


Forward Kinematics

- without proof I claim:

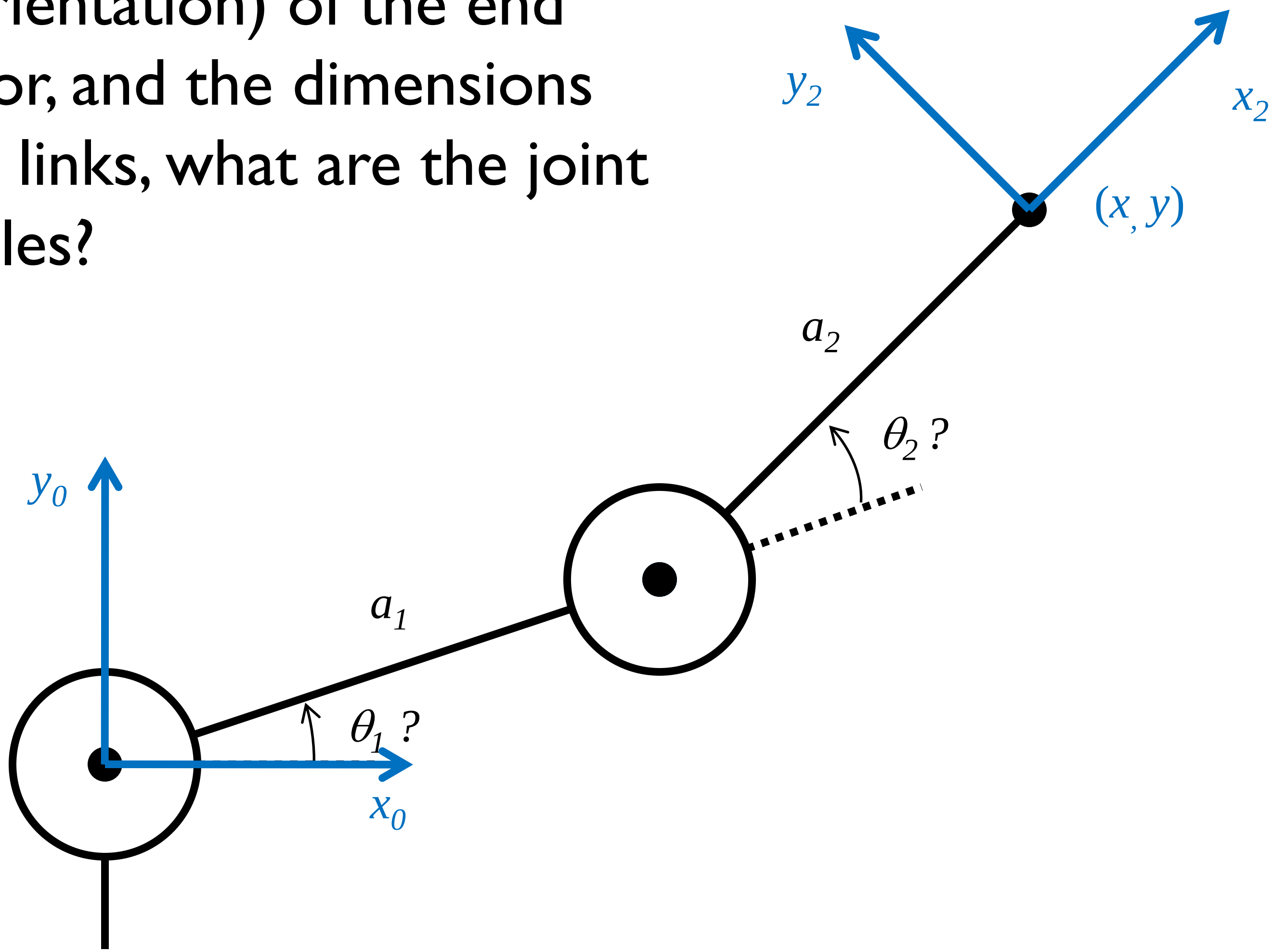
$x_2 = (\cos(\theta_1 + \theta_2),$
$\sin(\theta_1 + \theta_2))$

$y_2 = (-\sin(\theta_1 + \theta_2),$
$\cos(\theta_1 + \theta_2))$



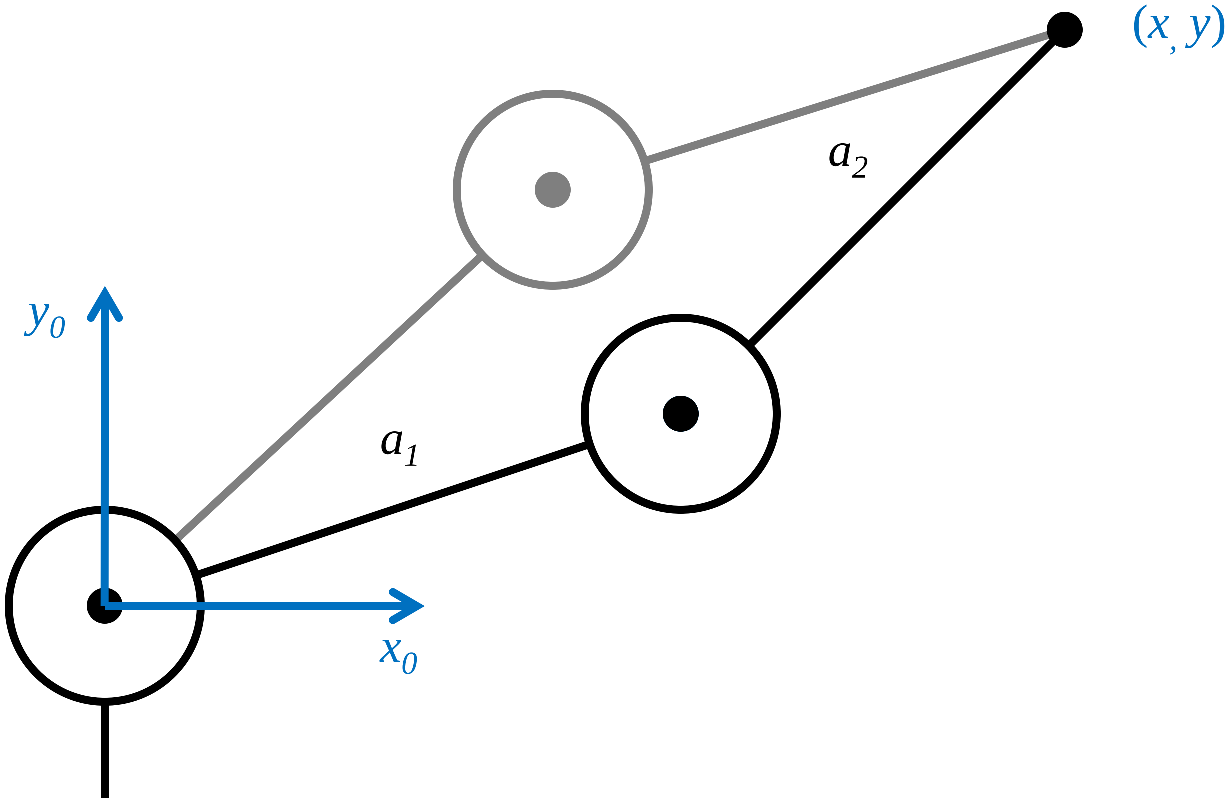
Inverse Kinematics

- given the position (and possibly the orientation) of the end effector, and the dimensions of the links, what are the joint variables?



Inverse Kinematics

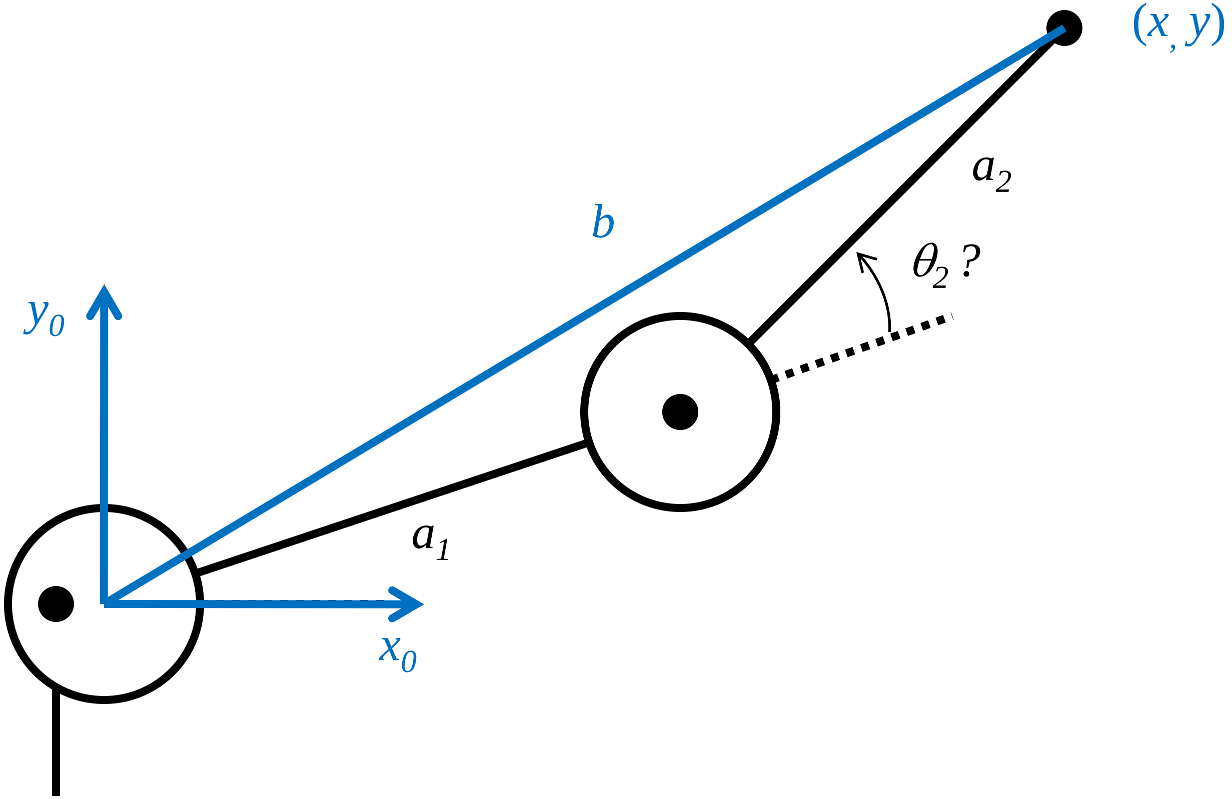
- harder than forward kinematics because there is often more than one possible solution



Inverse Kinematics

law of cosines

$$b^2 = a_1^2 + a_2^2 - 2a_1 a_2 \cos(\pi - \theta_2) = x^2 + y^2$$



Inverse Kinematics

$$-\cos(\pi-\theta_2)=\frac{x^2+y^2-a_1^2-a_2^2}{2a_1a_2}$$

and we have the trigonometric identity

$$-\cos(\pi-\theta_2)=\cos(\theta_2)$$

therefore,

$$\cos\theta_2=\frac{x^2+y^2-a_1^2-a_2^2}{2a_1a_2}=C_2$$

We could take the inverse cosine, but this gives only one of the two solutions.

Inverse Kinematics

Instead, use the two trigonometric identities:

$$\sin^2\theta + \cos^2\theta_2 = 1 \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

to obtain

$$\theta_2 = \tan^{-1}\frac{\pm\sqrt{1-C_2^2}}{C_2}$$

which yields both solutions for θ_2. In many programming languages you would use the four quadrant inverse tangent function `atan2`

```
c2 = (x*x + y*y - a1*a1 - a2*a2) / (2*a1*a2);
s2 = sqrt(1 - c2*c2);
theta21 = atan2(s2, c2);
theta22 = atan2(-s2, c2);
```

Inverse Kinematics

- Exercise for the student: show that

$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{a_2 \sin\theta_2}{a_1 + a_2\cos\theta_2}\right)$$